Question 1 (12 marks) Begin a new booklet **Marks**

(a) Factorise fully $16x^3 - 2$ **2**

(b) Consider the points $A(-3,2)$ and $B(6,-4)$. Find the coordinates of the point $P(x, y)$ that divides the interval AB in the ratio of 2:1. **2**

(c) Solve the inequality $\dfrac{2}{x+1} < 1$. **2**

(d) Consider $f(x) = 3x^3 + 6$. Explain, using calculus, why $f(x)$ is always increasing. **2**

(e) Differentiate:

(i) $x\cos^2 3x$ **2**

(ii) $x^2 \tan^{-1} x$ **2**

Question 2 (12 marks) Begin a new booklet **Marks**

(a) Find the values of k such that $(x-2)$ is a factor of the polynomial **2**
$P(x) = x^3 - 2x^2 + kx + k^2$.

(b) Find correct to the nearest degree the acute angle between the lines **2**
$y = 3x + 1$ and $x + y - 5 = 0$

(c) Evaluate $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}}$ **2**

(d) Use the substitution $t = \tan\dfrac{x}{2}$ to show that $\dfrac{1-\cos x}{\sin x} = \tan\dfrac{x}{2}$ **3**

(e) Graph $y = 2\sin 3x$ for $0 \le x \le 2\pi$ **3**

Question 3 (12 marks) Begin a new booklet **Marks**

(a) If $5x^3 - 6x^2 - 29x + 6 = 0$ has roots α, β, γ then find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **2**

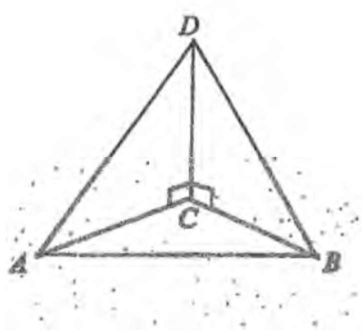
(b) Use Mathematical Induction to prove that $2^n \geq 1 + n$ for $n \geq 1$ **4**

(c) Use the substitution $u = x^2 + 1$ to evaluate $\int_1^7 \frac{x}{(1+x^2)^2} dx$. **4**

(d) Find the sum of the infinite series $\sin^2 x + \sin^4 x + \sin^6 x + \ldots$ Express your answer in simplest form. **2**

Question 4 (12 marks) Begin a new booklet **Marks**

(a)



Three points *A, B* and *C* lie on a plane. Points *A* and *B* are 30 metres apart and $\angle ACB = 120^\circ$. A vertical flagpole, *CD*, of height *h* metres stands at *C*. From *A* the angle of elevation of the top, *D,* of the flagpole is 30°. From *B* the angle of elevation to *D* is 45°

(i) Find the length of AC and BC in terms of *h*. **2**

(ii) Hence find the value of *h* to one decimal place. **2**

(b) The parametric equations of a parabola are

$x = 2t$

$y = 2t^2$

(i) Find the Cartesian equation of the parabola. **1**

(ii) Find the equation of the tangent to this parabola at the point where $x = 2$. **2**

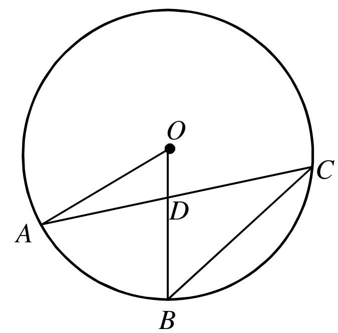
(c) Find $\int \sin^2 2x \, dx$ **2**

(d) (i) Find the domain and range of the function $f(x) = \cos^{-1}(2x)$. **2**

(ii) Sketch the graph of the curve $f(x) = \cos^{-1}(2x)$. **1**

Question 5 (12 marks) Begin a new booklet **Marks**

(a)



A, B and *C* are points lying on circle with centre *O*. *AO* is parallel to *BC*.
OB and *AC* intersect at *D*. $\angle ACB = 31^\circ$.

(i) Find the size of $\angle AOB$, giving reasons. **2**

(ii) Find the size of $\angle BDC$, giving reasons. **2**

(b) At time t years after the start of the year 2000, the number of individuals in A population is given by $N = 80 + Ae^{0.1t}$ for some constant $A > 0$.

(i) Show that $\dfrac{dN}{dt} = 0.1(N-80)$. **1**

(ii) If there were 100 individuals in the population at the start of the year 2000 Find the year in which the population size is expected to reach 200. **3**

(c) (i) Write $\sqrt{3}\cos x - \sin x$ in the form $r\cos(x+\alpha)$. **1**

(ii) Hence, give the general solution to $\sqrt{3}\cos x - \sin x = 1$ **3**

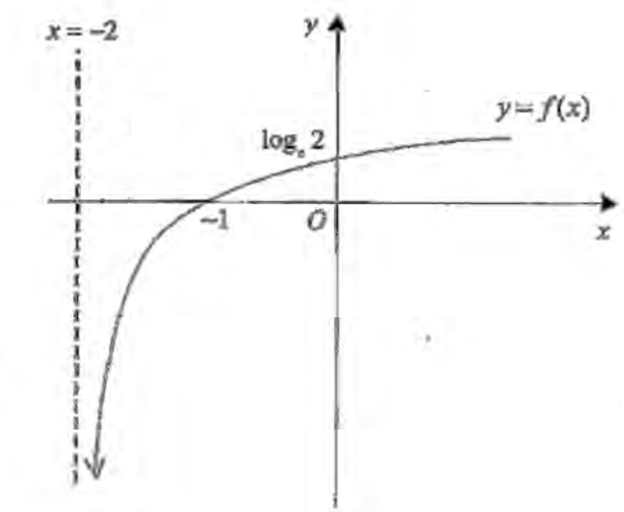
Question 6 (12 marks) Begin a new booklet **Marks**

(a) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O in the line, v ms^{-1} is given by $v = \dfrac{1}{x+1}$ and acceleration a ms^{-2}. Initially the particle is at O.

(i) Express a as a function of x. **1**

(ii) Express x as a function of t. **3**

(b)



The diagram shows the graph of the function $f(x) = \ln(x+2)$.

(i) Copy the diagram and on it draw the graph of the inverse function $f^{-1}(x)$ showing the intercepts on the axes and the equation of the asymptote. **2**

(ii) Show that the x-coordinates of the points of intersection of the curves $y = f(x)$ and $y = f^{-1}(x)$ satisfy the equation $e^x - x - 2 = 0$. **2**

(iii) Show that the equation $e^x - x - 2 = 0$ has a root α such that $1 < \alpha < 2$. **2**

(iv) Use one application of Newton's method with an initial approximation $\alpha_0 = 1.2$ to find the next approximation for the value of α, giving your answer correct to one decimal place. **2**

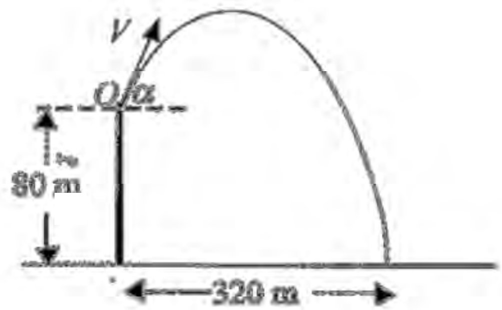
Question 7 (12 marks) Begin a new booklet **Marks**

(a) A particle is moving in a straight line with Simple Harmonic Motion. At time t seconds it has displacement x metres from a fixed point O on the line, given by $x = 1 + 3\cos\frac{t}{2}$, velocity v ms^{-1} and acceleration a ms^{-2}.

(i) Show that $a = -\frac{1}{4}(x-1)$. **2**

(ii) Find the distance travelled and the time taken by the particle over one complete oscillation of its motion. **2**

(c)



A particle is projected with speed V ms^{-1} at an angle α above the horizontal from a point O at the top edge of a vertical cliff which is 80 m above horizontal ground. The particle moves in a vertical plane under gravity where the acceleration due to gravity is $10\,\text{ms}^{-2}$. It reaches its greatest height after 3 seconds and hits the ground at a horizontal distance of 320 m from the foot of the cliff.

The horizontal and vertical displacements, x and y metres respectively, of the particle from the point O after t seconds are given by $x = Vt\cos\alpha$ and $y = -5t^2 + Vt\sin\alpha$. (Do NOT prove these results.)

(i) Show that $V\sin\alpha = 30$. **1**

(ii) Show that the particle hits the ground after 8 seconds. **2**

(iii) Show that $V\cos\alpha = 40$. **1**

(iv) Hence find the exact value of V and the value of α to the nearest minute. **2**

(v) Find the time after projection when the direction of motion of the particle first makes an angle of 45° below the horizontal. **2**

EXT SOLS

Q1 (a) $2(2x-1)(4x^2+2x+1)$ ②

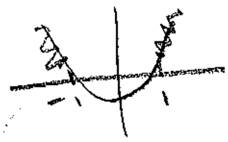
(b) $\dfrac{2(6)-3}{3}$, $\dfrac{-4(2)+2(1)}{3}$ ②

$P(3,-2)$

(c) $\dfrac{2}{x+1} < 1$

$2x+2 < x^2+2x+1$

$0 < x^2-1$ ②



$x < -1$ or $x > 1$

(d) $f'(x) = 27x^2$ and $x^2 \geq 0$

so $27x^2$ is always ≥ 0 ②

$\therefore$ pos. gradient

always increasing

(e) (i) $u = x \qquad v = \cos^2 3x$

$u' = 1 \qquad v' = -6\cos 3x \sin 3x$

$\therefore \dfrac{d}{dx}\left(x\cos^2 3x\right) = \cos^2 3x - 6x\cos 3x \sin 3x$ ②

(ii) $u = x^2 \qquad v = \tan^{-1} x$

$u' = 2x \qquad v' = \dfrac{1}{1+x^2}$

$2x\tan^{-1}x + \dfrac{x^2}{1+x^2}$ ②

Q2

(a) $P(2) = 2^3 - 2(2)^2 + k(2) + k^2$

$0 = 8 - 8 + 2k + k^2$

$0 = k(2+k)$

$k = 0, -2.$

(2)

(b) $y = 3x+1 \quad m_1 = 3$

$y = -x + 5 = 0 \quad m_2 = -1$

$\tan\theta = \left|\dfrac{3-(-1)}{1 \times 3(-1)}\right| = 2$

$\therefore \theta \doteqdot 63^\circ$

(2)

(c) $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}} = \left[\sin^{-1}\frac{x}{2}\right]_0^1$

$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$

$= \dfrac{\pi}{6} - 0$

$= \dfrac{\pi}{6}$

(2)

(d) $\left(1 - \dfrac{1-t^2}{1+t^2}\right) \div \dfrac{2t}{1+t^2}$ (1).

$\dfrac{1+t^2-1+t^2}{\cancel{1+t^2}} \times \dfrac{\cancel{1+t^2}}{2t}$ (1)

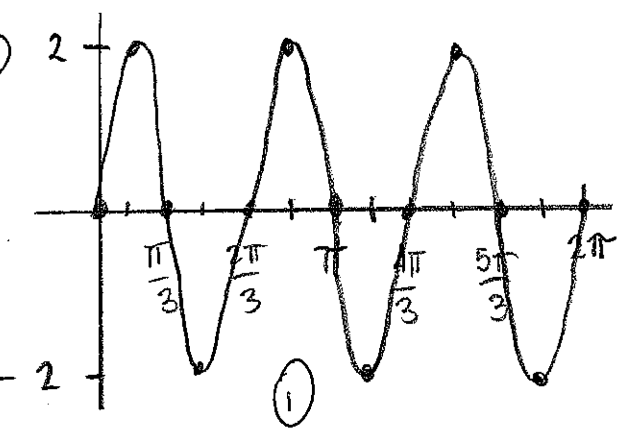
$\dfrac{2t^2}{2t} = t$

but $t = \tan\dfrac{x}{2}$ (1)

Total (3)

(e) $y = 2\sin 3x$ $\qquad \dfrac{2\pi}{3} = \text{per}$ (1)

(1)



(1)

Q3

(a) $\alpha + \beta + \gamma = \frac{-b}{a}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$

$\alpha\beta\gamma = \frac{-d}{a}$

$\frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{\frac{c}{a}}{\frac{-d}{a}}$

$\frac{c}{a} \times \frac{a}{-d}$

$-\frac{29}{\cancel{5}} \times \frac{\cancel{5}}{-6} = \frac{29}{6}$

(2)

d) $S_\infty = \frac{a}{1-r}$

$= \frac{\sin^2 x}{1-\sin^2 x}$

$= \frac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$ 2

(b) Show true for $n=1$

$2^1 \geq 1+1$

$2 \geq 2$ ✓

$\therefore$ true for $n=1$

assume true for $n=k$

$2^k \geq 1+k$

Prove true for $n=k+1$

LHS $= 2^{k+1}$ RHS $= k+1+1$

$= k+2$

$2^k \cdot 2$

but $2^k \geq 1+k$ if $k \geq 1$

$\therefore 2 \times 2^k \geq 2(1+k)$ which is greater than $k+2$

$= 2+2k$

(4)

c) $u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\therefore \frac{1}{2}\int_2^{50} \frac{du}{u^2} = -\frac{1}{2}\left[\frac{1}{u}\right]_2^{50}$

$= -\frac{1}{2}\left[\frac{1}{50} - \frac{1}{2}\right]$

$= 6/25 = 0.24$

(4)

4a) $\frac{AC}{h} = \tan 60°$

$AC = h \tan 60$

$AC = h\sqrt{3}$

$BC = h \tan 45°$

$BC = h$

$900 = 3h^2 + h^2 - 2\sqrt{3}h^2 \cos 120°$

$900 = h^2(3 + 1 - 2\sqrt{3} \times -1/2)$

$900 = h^2(5.732\ldots)$

$157.01\ldots = h^2$

$h \doteq 12.53$

$\sqrt{3}\cos x - \sin x = r(\cos x \cos\alpha - \sin\alpha \sin x)$

$r = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{4} = 2$

$2\cos\alpha = \sqrt{3}$ $\quad$ $2\sin\alpha = 1$

$\cos\alpha = \frac{\sqrt{3}}{2}$ $\quad$ $\sin\alpha = 1/2$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = 30°$

$2\cos(x + 30°) = 1$

$\cos(x + 30°) = 1/2$

$\cos(x + 30°) = \cos(60°)$

$x + 30 = 360n \pm 60°$

$x = 360n + 30°$ or $360n - 90°$

4b) $x = 2t$

$y = 2t^2$

(i) $\frac{x}{2} = t$

$\therefore \quad y = 2\left(\frac{x}{2}\right)^2$

$y = 2\left(\frac{x^2}{4}\right)$

$y = \frac{x^2}{2}$ (1)

$2y = x^2.$

(ii) $y' = \frac{1}{2} \times 2x$

$= x$

@ $x = 2 \quad m_t = 2$

@ $x = 2 \quad y = \cancel{8}\,2$

$\therefore$ equation of tangent is $y - 2 = 2(x-2)$

$y - 2 = 2x - 4$ (2)

$y = 2x - 2.$

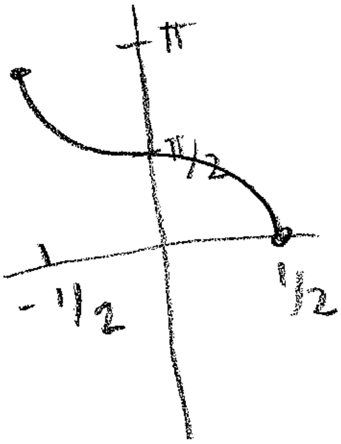
(c) $\int \sin^2 2x \, dx$

$= \frac{1}{2}\int (1 - \cos 4x)\, dx$

$= \frac{1}{2}x - \frac{1}{2} \times \frac{1}{2}\sin 4x + c$

$= \frac{1}{2}x - \frac{1}{4}\sin 4x + c$ (2)

Q4) d) (ii)



①

(i) D : $-\frac{1}{2} \le x \le \frac{1}{2}$

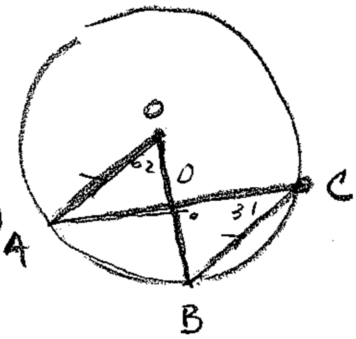
R : $0 \le y \le \pi$. ②

Q5) a) (i) $\angle AOB = 62°$ angle at the centre is twice the angle on circumference standing on the same arc.

②

(ii) $\angle CBD = 62$ (alt angles bet parallel lines)

and ∴ $\angle CDB = 87°$ (angle sum of Δ)



(b) (i) $N = 80 + Ae^{0.1t}$

$\frac{dN}{dt} = 0.1Ae^{0.1t}$ ①

$= 0.1(N-80)$

(ii) $N = 100$ when $t = 0$ ∴ $A = 20$

$200 = 80 + 20e^{0.1t}$

$e^{0.1t} = 6$

$0.1t = \ln 6$

$t = 10 \ln 6$

∴ $t = 17.92$

So in 2017 the pop'n reaches 200.

Q6)

a i) $v = \frac{1}{x+1}$

$\frac{d}{dx}\left(\frac{1}{2(x+1)^2}\right) = a$

$= \frac{-1}{(x+1)^3}$.

(ii) $\frac{dx}{dt} = \frac{1}{x+1}$

$\therefore \frac{dt}{dx} = x+1$

$t = \int (x+1)\,dx$

$= x^2 + x + c$

when $t=0$ $x=0$

$\therefore c = 0$

$t = x^2 + x$

$\cancel{t = x^2}$

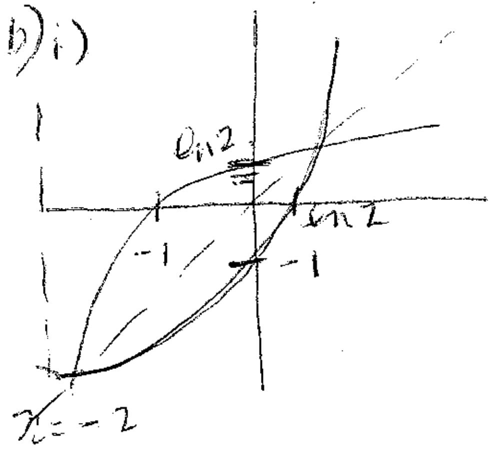
$t + \frac{1}{4} = x^2 + x + \frac{1}{4}$

$t + \frac{1}{4} = (x + \frac{1}{2})^2$

$\pm\sqrt{t + \frac{1}{4}} = x + \frac{1}{2}$

$\therefore -\frac{1}{2} \pm \sqrt{t + 1/4} = x$

b) i)



2

(ii) intersect where $y = x$

$\ln(x+2) = x$

$x + 2 = e^x$

$e^x - x - 2 = 0$ ✓ 2

(iii) $g(x) = e^x - x - 2$

$g(1) = e - 3 < 0$ and

$g(2) = e^2 - 4 > 0$

$\therefore 1 < \alpha < 2$ where α is a root.

(iv) $g'(x) = e^x - 1$

$1.2 - \frac{e^{1.2} - 3.2}{e^{1.2} - 1} \doteqdot 1.1$

2

7a)

(i) $x = 1 + 3\cos\frac{t}{2}$, V m/sec a m/sec

$$v = \frac{dx}{dt} = -\frac{3}{2}\sin\frac{t}{2}$$

$$a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{-3}{4}\cos\frac{t}{2}$$

but $x - 1 = 3\cos\frac{t}{2}$

2

$$\therefore\ a = -\frac{1}{4}(x-1)\ \checkmark$$

(ii) $v = 0$ when $-\frac{3}{2}\sin\frac{t}{2} = 0$

at rest $\sin\frac{t}{2} = 0$

$t = 0$, $t = 2\pi$, $t = 4\pi$ . . .

$\therefore$ when $t = 0$ $x = 1 + 3 = 4$

$t = 2\pi$ $x = 1 - 3 = -2$

$\therefore$ from 0 and back is 12 m.

and it takes 4π sec

period $\frac{2\pi}{1/2} = 4\pi$

1b.

(i) $\dot{y} = -10t + V\sin\alpha$ and $\dot{y} = 0$ when $t = 3$ (1)

$0 = -30 + V\sin\alpha$

$\therefore V\sin\alpha = 30$.

(ii) $y = -80 = -5t^2 + 30t$ (2)

$\therefore t^2 - 6t - 16 = 0 \quad (t \geq 0)$

$(t-8)(t+2) = 0 \quad \therefore t = 8$.

(iii) $x = 320$ when $t = 8$ (1)

$320 = 8V\cos\alpha$

$\therefore V\cos\alpha = 40$

(iv) $V^2(\sin^2\alpha + \cos^2\alpha) = 30^2 + 40^2$ (2)

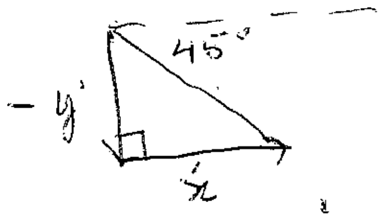
$V^2 = 2500$

$\therefore V = 50$

$\dfrac{V\sin\alpha}{V\cos\alpha} = \dfrac{30}{40} \quad \therefore \tan\alpha = \dfrac{3}{4}$

and $\alpha = 36°52'$

(v) (2)



$\dot{y} = -\dot{x}$

$-10t + 30 = -40$

$\therefore 10t = 70$

$t = 7$

hence after 7 seconds